Mixed Finite Element Solution of Helical Beams With Variable Cross-Section Under Arbitrary Loading †

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ABSTRACT

In this study, helical beams with variable cross-section under arbitrary loading is analyzed by finite elements. The proposed method allows for all types of boundary conditions to be easily taken into account. With the developed helical finite element HL, any arbitrary space structure can be solved. In this study shear effects have also been taken into consideration. The presented HL element has two nodes with 2x12 degrees of freedom and linear interpolation functions are used in solution.

1. INTRODUCTION

Helical beams are defined as beams circular in plan view and generally supported at their two ends. They are three-dimensional structures. The analysis of continuous helicoidal girders is generally found to be a laborious and cumbersome task. The increasing architectural use of free standing helicoidal stairs has created a need for a simple but thorough analysis of forces and moments on such structures, besides displacements and rotations obtained in critical locations on them, for design purposes.

Using the Gateaux differential, a functional for helicoidal beams is generated. With variational methods a mixed type finite element is obtained. Forces and moments are the unknowns of the element outside of displacements and rotations. Using the advantages of the proposed technique, helicoidal structures of any boundary or loading conditions are solved by the use of the sophisticated HL element. Precision of results with regards moments or forces are nearly at the same level as the displacements and rotations and convergence of results with respect to the number of unknowns is satisfactory.

2. THE FUNCTIONAL

Using the functional analysis method the following functional is obtained

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\[ I(\tilde{y}) = [-\tilde{u}, \frac{d\tilde{T}}{ds}] + [\tilde{r}x, \tilde{T}] - [\tilde{q}, \tilde{u}][\tilde{m}, \tilde{\Omega}] - \left[ \frac{d\tilde{M}}{ds}, \tilde{\Omega} \right] \frac{1}{2} [\tilde{D}^{-1}, \tilde{M}] - \frac{1}{2} [\tilde{C}^{-1}, \tilde{T}, \tilde{T}] + [\tilde{u}, \tilde{T}]_e + [\tilde{\Omega}, \tilde{M}]_e + [(\tilde{T} - \hat{T}), u]_\sigma + [(\hat{M} - \hat{\tilde{M}}), \tilde{\Omega}]_\sigma \]  

(1)

\(\tilde{u}, \tilde{\Omega}, \tilde{T}, \tilde{M}, \tilde{m}, \tilde{q}\) are displacement, rotation, force, moment, distributed external moment and external distributed force vectors respectively. The subscripts \(e\) and \(\sigma\) represent the geometric and dynamic boundary conditions. The shearing and bending rigidities are defined as:

\[
\tilde{C}^{-1} = \begin{bmatrix}
k'/GA & 0 & 0 \\
0 & k'/GA & 0 \\
0 & 0 & 1/EA
\end{bmatrix}; \quad \tilde{D}^{-1} = \begin{bmatrix}
1/EI_n & 0 & 0 \\
0 & 1/EI_b & 0 \\
0 & 0 & 1/EI_t
\end{bmatrix}
\]  

(2)

Displacement, rotation, force and moment vectors with their positive directions are shown in Figure 1 with respect to Frenet coordinates. For the finite element formulation linear interpolation functions are used. Since the internal stresses are not uniform, the aim of engineering is to reduce the size of the cross-section to be able to satisfy the economic requirements, and therefore the necessity for a variable cross-section is indicated. The variable cross-section property is introduced into the finite element matrix without spoiling the simplicity of the HL element matrix.

![Figure 1. Positive displacement, rotation, force, and moment components with respect to t, n, b axes.](image)

3. CONCLUSION

Since the functional contains only first order derivatives, linear interpolation functions are used for the development of the mixed finite element HL. Using this element, helicoidal space beams with variable cross-sections and under arbitrary distributive loads, having any boundary condition can be solved quite easily.

Using this element test examples are solved and from the point of view of engineering precision, convergence of results obtained to exact results proved the
efficiency of the HL element. A variable cross-section problem and a conical helix are analyzed as further examples and the results of all are found reasonably convergent.

In summary, the HL element is applicable to curved space bars of constant or variable cross-section with any boundary conditions. The presented elegant finite element matrix is very efficient since it is simple and convergence of results with this method is quickly achieved as predicted.

References


