Mixed Finite Element Analysis of Hyperbolic Paraboloid Shells Using Gâteaux Differential

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ABSTRACT

An isoparametric rectangular mixed finite element is developed for the analysis of hypars. The theory of shallow thin hyperbolic paraboloid shells is based on Gâteaux differential. This functional is written in operator form and is shown to be potential. Proper dynamic and geometric boundary conditions are obtained. Applying variational methods to this functional, the HYP9 finite element matrix is obtained in an explicit form. Since only first order derivatives exist in the functional, bilinear shape functions are used and a C° class conforming shell element is presented. The formulation is applicable to any boundary and loading condition. The HYP9 element has four nodes with nine degrees of freedom (DOF) per node-three displacements, three inplane forces and two bending, one torsional moment (4 × 9). The performance of this simple, and elegant shell element, is verified by applying it to some test problems existing in the literature. Since element matrix is obtained explicitly, there is an important save of computer time.

1. Introduction

An isoparametric rectangular mixed finite element is developed for the analysis of hypars. The theory of shallow thin hyperbolic paraboloid shells is based on Kirchhoff-Love's hypothesis and a new functional is obtained using the Gâteaux differential. This functional is written in operator form and is shown to be potential. Proper dynamic and geometric boundary condition are obtained. Applying variational methods to this functional, the HYP9 finite element matrix is obtained in an explicit form. Since only first order derivatives exist in the functional, linear shape functions are used and a C° conforming shell element is presented. The formulation is applicable

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to any boundary and loading condition. The HYP9 element has four nodes with nine degrees of freedom (DOF) per node-three displacements, three forces and two bending, one torsional moment \((4 \times 9)\). The performance of this simple, and elegant shell element, is verified by applying it to some test problems existing in the literature. Since element matrix is obtained explicitly, there is an important saving in computer time. Functional for the shallow hyperbolic paraboloid thin shell is,

\[
I_h(y) = -[u_y, Q] - [u_x, P] - [v_y, N] - [v_x, Q] + \\
\frac{1}{R_{12}} \{ -2(Q, w) + [M_y, w] + [T_y, v] + [K_{x, v}] + [T_{x, u}] \} \\
- [K_{x, w}, w] - [M_y, w] - [T_{x, w}, w] - [T_{x, w}, w] + \frac{1}{2B(1 - \mu^2)} \{ [P, P] + [N, N] - 2\mu [P, N] \} \\
+ \frac{1}{2D(1 - \mu^2)} \{ [K, K] + [M, M] - 2\mu [K, M] \} + \frac{1}{D(1 - \mu)} [T, T] \\
+ [q_x, u] + [q_y, v] + [q_x, w] + \left\{ \left[ \left( w_y - \frac{1}{R_{12}} u \right), (M - M) \right] \right\} \\
+ \left[ \left( w_y - \frac{1}{R_{12}} v \right), (K, \hat{K}) \right] + \left[ \left( w_x - \frac{1}{R_{12}} v \right), (T - \hat{T}) \right] \\
+ \left[ \left( w_y - \frac{1}{R_{12}} u \right), (T - \hat{T}) \right] + [u, \hat{Q}] + [v, \hat{Q}] + [u, \hat{P}] + [v, \hat{N}] \\
+ [w(M_y + \hat{K}_{x} + \hat{T}_{x} + \hat{T}_{y})] + \{ [(Q + P), (u - \hat{u})] + [(Q + N), (v - \hat{v})] \} \\
+ [M_x, (w - \hat{w})] + [K_{x}, (w - \hat{w})] + [T_{x}, (w - \hat{w})] + [T_{y}, (w - \hat{w})] \\
+ \left[ \left( \hat{w}_y - \frac{1}{R_{12}} \hat{u} \right), (M + T) \right] + \left[ \left( \hat{w}_x - \frac{1}{R_{12}} \hat{v} \right), (K + T) \right] \right\}_{\epsilon}
\]

(1)

It is believed that the Gâteaux differential is a powerful method and has many advantages over Hellinger-Reissner’s (HR) principles. In derivation of the functional with HR principles:

- For the problem under consideration, a functional assumes a stationary value, and the boundary condition (BC) terms must be known at the beginning,
- BC terms are included into the functional by Lagrange multipliers,
- Through that procedure choice of the parameters and sign of the BC terms may be different.

On the other hand using the Gâteaux differential method:
• It can be applied to any field equation for which a stationary functional is not known beforehand,
• Field equations must be tested to see if it is potential,
• BC terms are constructed and included to the functional in a rational way,
• It is obvious that if true terms can be choosen in HR principle, the resultant functional will coincide with the one which is derived by the Gâteaux differential.

Other superiorities of the suggested technique in finite element formulation can be summarized as:

1). Results are precise because;

a) Forces and moments are the necessary unknowns in an engineering problem and by using the proposed technique they are calculated independently of the displacements. For example, in assumed displacement model, forces and moments are calculated from derivatives of the displacements and if there are sharp changes in the slopes of the unknowns, a fine meshing is needed besides considering the aspect ratios,

b) The HYP9 element matrix is obtained in an explicit form so there is no need for numerical integrations,

c) Using the comfort of mathematics, the functional is obtained along with the necessary BC terms.

2). Computer time is not wasted, because;

a) The HYP9 matrix is given in an explicit form hence there is no need for a coefficient matrix calculation. This point is very important especially when different element sizes are needed in a problem. Also the element matrix consists of very simple terms and there are no numerical integrations,

b) For engineers, forces and moments are the necessary unknowns for design purposes and at the same time they are the basic unknowns of the HYP9 element besides the displacements hence there is no need for a back substitution process by this method.

As a consequence, to the contrary of its simplicity, the HYP9 element is a sophisticated, fast converging, reliable shell element.

References


