Mixed Finite Element Solution of Variable Cross-Sectional Hyperbolic Paraboloid Shells

Mehmet H. OMURTAG*  
A. Yalçın AKÖZ**

ABSTRACT

HYP9 finite element matrix is given in an explicit form by Omurtag-Aköz [1,2]. Since only first order derivatives exist in the functional, linear shape functions are used and a $C^0$ conforming shell element is presented. In this present study variation of the thickness is also included into the formulation without spoiling the simplicity of the HYP9. The formulation is applicable to any boundary and loading condition. The HYP9 element has four nodes with nine degrees of freedom per node-three displacements, three inplane forces and two bending, one torsional moment ($4 \times 9$). Since element matrix is obtained explicitly, there is an important save of computer time.

1. Introduction

The rectangular finite element matrix HYP9 is given in an explicit form by Omurtag-Aköz [1]. Since only first order derivatives exist in the functional, linear shape functions

$$i^{(\xi,\eta)} = \frac{1}{4} (1 + \xi_i)(1 + \eta_i)$$

are used and a $C^0$ conforming shell element is presented. In this study variation of the thickness (see Figure 1) is also included into the formulation without spoiling the simplicity of the HYP9. If $\zeta$ is an arbitrary varying property (such as thickness), it can be defined on the element as

$$\zeta = \sum_{k=1}^{4} \zeta_k \psi_k$$

and variation of this property under the area integral $dA$ becomes,

$$[k_1]_\zeta = \int_A \left[ \sum_{k=1}^{4} \zeta_k \psi_k \right] m \psi n dA$$

where $m,n=1,...,4$. Explicit form of the submatrix given in Equation (3) is,

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* ITU Faculty of Civil Engineering, 80626 Maslak, İstanbul  
** ITU Faculty of Civil Engineering, 80626 Maslak, İstanbul  
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\[
[k_1]_\zeta = \begin{bmatrix}
9\zeta_1 + 3\zeta_2 & 3\zeta_1 + 3\zeta_2 & 3\zeta_1 + \zeta_2 & \zeta_1 + \zeta_2 \\
+3\zeta_3 + \zeta_4 & +\zeta_3 + \zeta_4 & +3\zeta_3 + \zeta_4 & +\zeta_3 + \zeta_4 \\
3\zeta_1 + 9\zeta_2 & \zeta_1 + \zeta_2 & \zeta_1 + 3\zeta_2 & \zeta_1 + 3\zeta_2 \\
+\zeta_3 + 3\zeta_4 & +\zeta_3 + \zeta & +\zeta_3 + 3\zeta & +\zeta_3 + 3\zeta \\
3\zeta_1 + \zeta_2 & \zeta_1 + \zeta_2 & \zeta_1 + 3\zeta_2 & \zeta_1 + 3\zeta_2 \\
+9\zeta_3 + 3\zeta & +3\zeta_3 + 3\zeta & +3\zeta_3 + 3\zeta & +3\zeta_3 + 9\zeta \\
\end{bmatrix}
\]

The formulation is applicable to any boundary (clamped, simply supported, diaphragm or free) and loading (point loading or distributed loading) condition. The \textit{HYP9} element has four nodes with nine degrees of freedom per node-three displacements \((u, v, w)\), three inplane forces \((P, N, Q)\) and two bending \((K, M)\), one torsional moment \((T)\)- as a total of 36 DOF \((4 \times 9)\) (See Figure 2). Since element matrix is obtained explicitly, there is an important saving in computer time.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Variable cross-sectional element. \textit{X-Y}: Global coordinates, \textit{\xi, \eta}: Local coordinates}
\end{figure}
Figure (2): Positive directions of displacements, forces and moments.

References